Computer Science 360
Final Examination
(Open Text Books and Notes)

Time: 3 hours December 15, 2007
Marks (Total 100)

10 1. Provide a deterministic finite automaton that accepts the language $(a^*b) \cup (b^*a)^*$.

10 2. For each of the following statements indicate whether it is true or false. Give a reason for each decision.

a. Every language accepted by a nondeterministic pushdown automaton can be accepted by a deterministic Turing Machine.

b. A finite automaton with three FIFO (first-in, first-out) queues is more powerful, in terms of ability to recognize languages, than a regular one tape deterministic Turing machine.

c. No problem can be both in NP and NP-complete.

d. P is a subset of the set of the NP-complete problems.

e. Backtracking is a method that can solve all problems in NP in exponential time.

12 3. Construct a pushdown automaton for the language over the alphabet $\Sigma = \{a, b, c\}$ which consists of all strings in Σ^* where the number of a's is equal to the sum of the number of b's and c's.

12 4. Give a divide-and-conquer algorithm to draw an approximation to the line segment connecting two points (x_1, y_1) and (x_2, y_2) by drawing points with only integer coordinates. Assume that x_1, y_1, x_2 and y_2 are integers.

14 5. Define the wacky-encoding, $E(w)$, of a string $w \in \{a, b, c\}^*$ as follows: $E(e) = e$, where e is the empty string, $E(a) = bb$, $E(b) = a$, $E(c) = c$ and if $w = du$ where $d \in \{a, b, c\}$ and $u \in \{a, b, c\}^*$ then $E(w) = E(d)E(u)$. For example, $E(acba) = bbabb$. Define the wacky-encoding of a language $L \subseteq \{a, b, c\}^*$, $E(L)$ to be the wacky-encoding of the strings in L.

Prove that if L is a language over the alphabet $\{a, b, c\}$ and L is accepted by a finite automaton, that then $E(L)$ is regular.

14 6. [Longest Decreasing Subsequence] Given a sequence of n integers, determine a subsequence (not necessarily contiguous) of maximum length in which the values in the subsequence form a strictly decreasing sequence. For example, in the sequence 1,8,-5,-4,-3,6,100,2,0 the maximum length decreasing subsequence is 8,6,2,0.

14 7. Give an efficient algorithm to find the cycle with the fewest edges in a given n vertex e edge undirected bipartite graph. If the given graph does not contain a cycle then that fact should be reported. If the graph does contain a cycle then the algorithm should output the vertices of the shortest cycle.

14 8. Consider a variation on the minimum spanning tree problem where you are given a connected undirected graph $G = (V, E)$ and three special edges from E, e_1, e_2 and e_3. Each edge of E has a nonnegative length. The problem is to find a subset T of the edges of G such that (i) T contains e_1, e_2 and e_3, (ii) the subgraph $G' = (V, T)$ is connected and (iii) the sum of the lengths of the edges in T is as small as possible.

a. Provide an efficient algorithm that will solve this problem

b. Prove that your algorithm is correct.